Mean-Field Statistical Theory of Finite Quantum Systems with Applications to Hot Dense Plasmas

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Recently, there has been a growing interest in statistical properties of the mesoscopic quantum systems, containing a large but finite number of interacting particles. There exist examples galore of such finite many-body systems in nature as well as in laboratory experiments, including ultracold Bose and Fermi gases confined in atomic traps, multinucleon nuclei and multielectron neutral atoms, nanoscale atomic clusters, and multielectron ions in hot dense plasmas. The number of coupled particles in such mesoscopic systems can vary from a few, as is the case with the bound electrons in low-Z atoms or ions, to hundreds of thousands for the case of trapped neutral alkali atoms. The range of relevant thermodynamic conditions is also very broad: from ultralow, cryogenic temperatures and low densities in the case of trapped Bose and Fermi gases to very high temperatures and densities in the case of some plasmas. Hence the development of a unified statistical theory of such systems, capable of taking into account finite size effects, is quite timely, and it can have a broad spectrum of potential applications. Since the number of particles is finite and fixed, we have to work with the canonical ensemble, which usually presents formidable difficulties.

Some progress along these lines has however been made to date [1–11]. Unfortunately, all these approaches suffer from significant drawbacks. In particular, some authors only consider either the systems with large numbers of particles [1–6, 9], or those at low temperatures [1–6], or they neglect the particle-particle interactions [1–5, 10, 11]. Other researchers treat the interactions phenomenologically, using a rather simplified statistical model [9]. Some alternative versions of the theory require an extensive amount of numerical work to obtain results for realistic systems [7, 8]. The other class of theories is only concerned with particles obeying classical statistics [12, 13]. Thus a problem of formulating a self-consistent statistical theory of finite systems, free of the just mentioned flaws, remains open.

The purpose of this report is to outline the highlights of such a theory. We follow the path integral approach to the generation of the statistical mean-field theory of Kerman and Levit, previously formulated for very large nuclear systems whose statistical behavior is specified by the grand canonical ensemble [14, 15]. We generalize this method to treat finite-size systems described by the canonical ensemble. We first develop the simplest possible variant of the mean-field theory that does not take into account the exchange effects, and we provide an exact closed form expression for the occupation number distribution of particles in finite quantum systems within the framework of the independent particle approximation. The derived occupation distribution function has simple analytical expressions in the important limiting cases of large and small numbers of particles in the system. The first case is relevant for trapped Bose and Fermi gases as well as nanoscale clusters, whereas the second one is of interest for low-Z atomic or plasma systems, having ions with a few bound electrons. We also show how our results can be straightforwardly generalized to include the exchange effects using the approach of Ref. [15]. Not only does our theory enable us to derive the canonical occupation number distribution in a self-consistent manner, but it treats Bose and Fermi systems within the same general framework as well.

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